Crossing regimes of temperature dependence in animal movement

Supporting Information

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I. Geometric similarity and the $M \propto L^3$ relationship

By assuming dynamic similarity, we are also tacitly assuming geometric similarity (Alexander, 1983, 2003; Biewener, 2003). Geometric similarity implies that for any organism of total length $l$, any limb of length $L$ will be linked to the total length of the organism through:

$$l = aL,$$

where $a$ is a constant (Biewener, 2003). Because the mass ($M$) of an organism is proportional to its volume ($V$), and the volume of a geometrically similar organism is proportional to the cube of its linear dimensions (i.e. $V \propto l^3$), then it follows that $M \propto l^3$. Using eq S.Ia we get:

$$M \propto (aL)^3,$$

which can be rewritten as $M \propto a^3L^3$ and because $a$ is a constant, it follows that $M \propto L^3$.

II. Temperature dependence of frequency for flapped wings

Before showing how to write the wingbeat frequency as a function of temperature, we will illustrate how dimensional analysis works with a simple example. Dimensional analysis is a common mathematical technique in physics that starts by postulating a hypothetical relationship between physical quantities, and then re-writes the whole as a function of the fundamental dimensions: mass, length and time. Finally, dimensional analysis involves finding how the initial physical quantities of interest may yield another quantity of known dimensions using some very basic algebra. In what follows we will work out the solution for the resonant frequency of limb movement derived from the inverted-pendulum model.
We start by postulating which physical quantities will be affecting the process of interest. In the inverted pendulum model, we have the weight of the organism, which is equal to the product of its mass (M) and the gravitational acceleration (g), and the length of the massless rod (L). So, if we are trying to find how these quantities affect the frequency of the pendulum we can postulate the following:

\[ f \propto M^\alpha g^\beta L^\gamma. \]  

\text{eq S.IIa}

Now, we rewrite the whole as a function of fundamental dimensions (M, L, T):

\[ T^{-1} \propto M^\alpha (LT^{-2})^\beta L^\gamma. \]  

\text{eq S.IIb}

The above equation comes from the fact that the dimensions of a frequency are 1/T, and those of an acceleration are L/T^2. The final step consists in finding the values of \( \alpha, \beta \) and \( \gamma \) that satisfy eq S.IIb. From inspecting eq S.IIb, it is easy to see that finding \( \alpha, \beta \) and \( \gamma \) that satisfy eq S.IIb is equivalent to solving the system of algebraic equations:

\[
\begin{align*}
\alpha &= 0 \\
\beta + \gamma &= 0 \\
-2\beta &= -1
\end{align*}
\]

\text{eq S.IIc}

because there is no dimension of mass in the left side of the equation (hence \( \alpha = 0 \)), there is no dimension of length in the left side of the equation (hence \( \beta + \gamma = 0 \)) and time goes as \( T^{-1} \) in the left side of the equation (hence \( -2\beta = -1 \)). The only solution to the above system is \( \alpha = 0 \) and \( \beta = -\gamma = 1/2 \). Thus,

\[ f \propto \frac{g}{\sqrt{L}}, \]

\text{eq S.IId}

as expected. Dimensional analysis is not only a powerful tool when no obvious model exists to infer the relationship between physical quantities, but often yields accurate results, as in this case. In the case of the inverted pendulum model, the actual
relationship between frequency and gravitational acceleration and length can be found in closed form solving a system of differential equations (Alexander, 1983), and thus dimensional analysis is not necessary. We used, however, this simple result to illustrate the overall approach and to show its power.

In what follows, we derive the temperature dependence of the frequency of wingbeats using the dimensional analysis. Assuming that wingbeats depend on the body weight of the animal \( (M^*g) \), its wingspan \( (b) \), its wing area \( (S) \) the density of the air \( (\rho) \), and then using dimensional analysis, it has been shown that the frequency at which winged animals can beat their wings can be written as (Pennycuick, 1990):

\[
f \propto \frac{M^{1/3} g^{1/2}}{b S^{1/4} \rho^{1/3}}. \quad \text{eq S.IIe}
\]

Assuming geometric similarity (see Supplementary Materials I), \( b \propto M^{1/3} \) because the wingspan is a length, and because \( S \propto L^2 \) and \( L \propto M^{1/3} \), then \( S \propto M^{2/3} \). Replacing into eq S.IIe we get that:

\[
f \propto \frac{g^{1/2} M^{1/3}}{\rho^{1/3} M^{1/3} M^{2/3}}, \quad \text{eq S.IIf}
\]

and after rearranging terms,

\[
f \propto \frac{g^{1/2}}{\rho^{1/3} M^{1/6}}. \quad \text{eq S.IIg}
\]

Using equation (4) of the main text \( M = \left( \frac{B}{b_0} \right)^4 \left( \frac{4 E_0}{4 \kappa_0 T} \right) \), and replacing into eq S.IIg it can be seen that:

\[
f \propto \sqrt[3]{\frac{g}{\rho}} \left( \frac{B}{b_0} \right)^{2/9} e^{\frac{2 E}{9 \kappa_0 T}} , \quad \text{eq S.IIh}
\]

as stated in the main text.
III. Temperature dependence of frequency for elastic appendages

Following Pennycuick’s work (1990), we postulated that the frequency at which an elastic, compliant massless appendage can move is a function of the gravitational acceleration \((g)\), the moment of inertia of the appendage \((I)\), its total length \((L)\), the spring (elastic) constant of the limb \((k)\), and the density of the air \((\rho)\):

\[
f \propto \rho^\alpha I^\beta g^\gamma L^\delta k^\varepsilon.
\]

eq S.IIIa

Dimensionally, eq S.IIIa becomes:

\[
T^{-1} \propto (ML^{-3})^\alpha (ML^2)^\beta (LT^{-2})^\gamma L^\delta (MT^{-2})^\varepsilon,
\]

eq S.IIIb

which yields the system,

\[
\begin{align*}
\alpha + \beta + \varepsilon &= 0 \\
-2\gamma - 2\varepsilon &= -1 \\
-3\alpha + 2\beta + \delta + \gamma &= 0
\end{align*}
\]

eq S.IIIc

The system is satisfied whenever:

\[
\begin{align*}
\varepsilon &= \frac{1}{2} - \gamma \\
\beta &= -\frac{1}{2} - \alpha + \gamma \\
\delta &= 1 + 5\alpha - 3\gamma
\end{align*}
\]

eq S.IIId

Because we have more variable than equations, the system is indeterminate. To find one possible solution, we impose some of the conditions already found by Pennycuick (1990), this is, that \(f\) goes with the square root of \(g\) and with the inverse of the cube root of \(\rho\) (see equation eq S.IIIh). So, imposing \(\alpha = -1/3\) and \(\gamma = 1/2\), we obtain \(\beta = 1/3\), \(\delta = -13/6\) and \(\varepsilon = 0\). Thus, eq S.IIIa becomes:

\[
f \propto \sqrt[3]{g} \frac{I^{1/3}}{\sqrt[3]{\rho} L^{13/6}}.
\]

eq S.IIIe
Rewriting $I$ as $ML^2$, and using geometric similarity, we can see that

$$f \propto \frac{\sqrt{g}}{\sqrt{\rho}} \frac{1}{M^{1/2}},$$

\text{eq S.IIIa}

just as eq S.IIg. Thus, the temperature dependence of the movement of elastic limbs is equivalent to that of wingbeats, as stated in the main text.

\textbf{IV. Temperature dependence of frequency for rubbed appendages}

For rubbed appendages, we postulated that the frequency of the movement of the appendages being rubbed against each other is a function of the force exerted on the appendage ($F$), its moment of inertia ($I$) as well as the length of the appendix ($L$). Then:

$$f \propto I^\alpha L^\beta F^\gamma,$$

\text{eq S.IVa}

where $F = \ddot{a} M$ and $\ddot{a}$ being the acceleration the muscles controlling the movement are able to exert on the appendage being moved. Dimensionally:

$$f \propto (ML^2)^\alpha (L^{-2}M)^\gamma,$$

\text{eq S.IVb}

which yields the following system

\[
\begin{align*}
\alpha + \gamma &= 0 \\
2\alpha + \beta + \gamma &= 0 \\
-2\gamma &= -1
\end{align*}
\]

\text{eq S.IVc}

The system is determinate and satisfied by $\alpha = -1/2$, $\beta = 1/2$ and $\gamma = 1/2$, which yields:

$$f \propto \frac{\sqrt{FL}}{I}.$$

\text{eq S.IVd}

Replacing $F$ by $\ddot{a} M$ and $I$ by $ML^2$, we get:

$$f \propto \sqrt{\ddot{a}} \frac{1}{\sqrt{L}}.$$

\text{eq S.IVe}
Assuming geometric similarity and substituting \( M = \left( \frac{B}{b_0} \right)^{\frac{d}{\epsilon}} e^{\frac{E_c}{\kappa_{int}}} \) we get:

\[
f \propto \sqrt{a} \left( \frac{B}{b_0} \right)^{\frac{d}{\epsilon}} e^{\frac{E_c}{\kappa_{int}}}, \quad \text{eq S.IVf}
\]
as stated in the main text.

V. Local stability analysis of consumer-resource model

To assess how temperature may affect the stability of consumer-resource interactions through its effect on the velocities of predators, we used local stability analysis. Given a dynamic model specified by a system of differential equations, a stability analysis consists in letting the system reach an equilibrium, and then impose a small (infinitesimal) perturbation that takes the system away from that equilibrium to assess whether it returns to the equilibrium or whether it goes away from it. To do so, we first need to analytically find such equilibrium, if it exists, and then calculate the matrix of derivatives of the system with respect to all state variables (here \( R \) and \( C \), see equation (11) of the main text), or Jacobian matrix. When evaluated at equilibrium, if the eigenvalues of the Jacobian are negative, the system will return to equilibrium after a small perturbation (i.e. the system is stable), and the system will go away from equilibrium otherwise (i.e. the system is unstable).

For the model specified in (eq 11), there are three possible equilibria, but only one where the two species persist, namely:

\[
R^* = \frac{d}{\alpha (\epsilon - d \eta)}, \quad \text{and} \quad \text{eq S.Va}
\]

\[
C^* = \frac{r \epsilon (K \alpha - dK \alpha \eta - d)}{K \alpha^2 (\epsilon - d \eta)^2}, \quad \text{eq S.Vb}
\]
The Jacobian matrix of the system is defined as:

$$
J_{k^*,c^*} = \begin{pmatrix}
\frac{\partial}{\partial R} \frac{dR}{dt} & \frac{\partial}{\partial C} \frac{dR}{dt} \\
\frac{\partial}{\partial R} \frac{dC}{dt} & \frac{\partial}{\partial C} \frac{dC}{dt}
\end{pmatrix}, \quad \text{eq S.Vc}
$$

and in this case it is equal to:

$$
J_{k^*,c^*} = \begin{pmatrix}
-r d \left[ \epsilon - K \epsilon \alpha \eta + d \eta (1 + K \alpha \eta) \right] & -\frac{d}{\epsilon} \\
K \alpha \epsilon (\epsilon - d \eta) & r \left( \frac{d}{K \alpha} + \epsilon - d \eta \right)
\end{pmatrix}. \quad \text{eq S.Vd}
$$

In all cases, $\alpha$ is to be defined as $\alpha(T) = A_d \sqrt{v_R^2 + \varphi_0^{\frac{B}{p_0}}} e^{-\frac{E}{kT}}$ (see main text), thus, the eigenvalues of the matrix are a function of temperature. Using the Mathematica code below, we can plot the leading eigenvalue as a function of temperature to make figure 4 of the main text.
VI. Cutoff temperatures and estimated activation energies

**Table SI:** Activation energy estimates and temperature cutoff values for all cursorial movements. OLS stands for Ordinary Least Squares and BP stands for Breakpoint regression.

<table>
<thead>
<tr>
<th>Common name</th>
<th>Inverted-pendulum (OLS)</th>
<th>Inverted-pendulum (BP High)</th>
<th>Inverted-pendulum (BP Low)</th>
<th>Spring-mass (OLS)</th>
<th>Spring-mass (BP High)</th>
<th>Spring-mass (BP Low)</th>
<th>Temperature cutoff (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viviparous lizard</td>
<td>1.89</td>
<td>10.71</td>
<td>0.99</td>
<td>0.63</td>
<td>3.57</td>
<td>0.33</td>
<td>9.1</td>
</tr>
<tr>
<td>Fiddler crab</td>
<td>2.06</td>
<td>2.76</td>
<td>0.86</td>
<td>0.68</td>
<td>0.92</td>
<td>0.30</td>
<td>22.4</td>
</tr>
<tr>
<td>American alligator</td>
<td>1.08</td>
<td>2.56</td>
<td>0.06</td>
<td>0.36</td>
<td>0.85</td>
<td>0.02</td>
<td>23.2</td>
</tr>
<tr>
<td>Dolichorinid ant</td>
<td>1.16</td>
<td>2.04</td>
<td>1.05</td>
<td>0.39</td>
<td>0.68</td>
<td>0.35</td>
<td>16.4</td>
</tr>
<tr>
<td>Green day gecko</td>
<td>0.65</td>
<td>0.90</td>
<td>0.38</td>
<td>0.22</td>
<td>0.13</td>
<td>0.30</td>
<td>24.8</td>
</tr>
<tr>
<td>Fence lizard</td>
<td>3.09</td>
<td>4.36</td>
<td>0.02</td>
<td>1.03</td>
<td>1.45</td>
<td>0.01</td>
<td>29.5</td>
</tr>
<tr>
<td>House cricket</td>
<td>1.59</td>
<td>2.93</td>
<td>0.70</td>
<td>0.53</td>
<td>0.98</td>
<td>0.23</td>
<td>25.71</td>
</tr>
<tr>
<td>House fly</td>
<td>0.91</td>
<td>–</td>
<td>–</td>
<td>0.30</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Desert iguana</td>
<td>3.17</td>
<td>14.80</td>
<td>0.73</td>
<td>1.06</td>
<td>4.93</td>
<td>0.24</td>
<td>21.4</td>
</tr>
<tr>
<td>Desert night lizard</td>
<td>2.10</td>
<td>4.47</td>
<td>0.42</td>
<td>0.70</td>
<td>0.49</td>
<td>0.14</td>
<td>22.14</td>
</tr>
<tr>
<td>Blue-tailed skink</td>
<td>2.01</td>
<td>8.56</td>
<td>1.03</td>
<td>0.67</td>
<td>2.85</td>
<td>0.34</td>
<td>21.4</td>
</tr>
<tr>
<td>Ornate Box turtle</td>
<td>0.78</td>
<td>1.46</td>
<td>-0.26</td>
<td>0.26</td>
<td>0.49</td>
<td>-0.09</td>
<td>31.43</td>
</tr>
</tbody>
</table>
Table S2: Activation energy estimates and temperature cutoff values for all other movements and appropriate models (see Table 3 in the main text).

<table>
<thead>
<tr>
<th>Common name</th>
<th>OLS</th>
<th>Breakpoint High</th>
<th>Breakpoint Low</th>
<th>Temperature cutoff (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fruit fly</td>
<td>0.78</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Chirping cricket</td>
<td>0.95</td>
<td>1.39</td>
<td>0.34</td>
<td>23.4</td>
</tr>
<tr>
<td>Calopterygid damselfly</td>
<td>1.68</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Garter snake</td>
<td>1.80</td>
<td>2.43</td>
<td>0.59</td>
<td>24.0</td>
</tr>
<tr>
<td>Predatory protist</td>
<td>0.49</td>
<td>0.90</td>
<td>0.83</td>
<td>18.8</td>
</tr>
<tr>
<td>Rattlesnake</td>
<td>2.01</td>
<td>3.77</td>
<td>1.22</td>
<td>17.4</td>
</tr>
</tbody>
</table>
VII. Fig SI-a and b

![Graph a](image1)

![Graph b](image2)

**Fig SI:** Fitted activation energies before breakpoint (low Temp, blue), and after the breakpoint (warm Temp, red), for the inverted-pendulum model (a), and the spring-mass model (b).

**Literature Cited**


